Stationary Disturbances in Periodically Modulated Rotating Disk Boundary Layers

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DiPaRT
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Dissecting the title...

**Periodically Modulated Rotating Disk Boundary Layers**

- Lengths scaled on constant boundary layer thickness: \( \delta = \sqrt{\nu/\Omega_0} \)
- Convectively unstable for stationary disturbances:
  - Type I: \( R_c \approx 286 \)
  - Type II: \( R_c \approx 440 \)

Malik (1986)
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Adding oscillation to *channel* flow can be stabilising

*Thomas et. al. (2011)*
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Periodically Modulated Rotating Disk Boundary Layers
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Dominant behaviour is Stokes layer for high-frequency, low amplitude oscillations.
Why rotating disks?

- Rotating disks $\approx$ Swept wings
Crossflow instabilities

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- Chemical applications - hydrodynamic voltammetry
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Why rotating disks?

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  - *Suppression of crossflow instability - focus on stationary disturbances.*

- Chemical applications - hydrodynamic voltammetry
- Other applications - mixing, atmospheric, oceanic
• Three-dimensional base flow

\[ \mathbf{U}_B = (U, V, W) \]
Adding the oscillation...

- Three-dimensional base flow

\[ \mathbf{U}_B = (U, V, W) \]

- Boundary conditions:

\[ V(r, z = 0, t) = r \Omega(t) = r (\Omega_0 + \epsilon \phi \cos(\phi t)) \]

\( \Omega_0 \) - constant rotation rate
\( \epsilon \) - angular displacement
\( \phi \) - oscillation frequency
Scalings

- Retain steady scalings:

\[ \delta = \sqrt{\frac{\nu}{\Omega_0}}, \quad \tau = (r_L \Omega_0) t \]
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  \[ R_k = r_L, \quad R_s = \epsilon \sqrt{\varphi} R_k \]

\[ \varphi = \frac{\phi}{\Omega_0} \] - number of oscillations per disk rotation period
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• Similarity structure:

\[ U_B = \left( \frac{r}{R_k} F, \frac{r}{R_k} G, \frac{1}{R_k} H \right) \]

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- Boundary conditions:
  \[ G(z = 0, \tau) = 1 + \frac{R_s \sqrt{\varphi}}{R_k} \cos \left( \frac{\varphi}{R_k} \tau \right) \]

\[ \varphi = \frac{\phi}{\Omega_0} - \text{number of oscillations per disk rotation period} \]
Scalings - Important Parts

- Boundary conditions:

\[ G(z = 0, t) = 1 + \frac{R_s \sqrt{\varphi}}{R_k} \cos \left( \frac{\varphi}{R_k} \tau \right) \]

\[ \Rightarrow U_w = : U_w \]

- Three parameters:

\( (R_k, R_s, \varphi) \) \quad or \quad \( (R_k, U_w, \varphi) \)

\( (R_s, U_w \to 0 \text{ recovers steady case}) \)
Scalings - Important Parts

- Boundary conditions:
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  \[ \equiv U_w \]

- Three parameters:
  \((R_k, R_s, \varphi)\) or \((R_k, U_w, \varphi)\)

  \((R_s, U_w \to 0 \text{ recovers steady case})\)

- Choose to vary:
  \((R_k, U_w, \varphi)\)

  and constrain \(U_w < 0.25\).
Approaches

- Three approaches:
  - Floquet Theory
  - Linear DNS
  - Solve Navier-Stokes equations using velocity-vorticity formulation.
Approaches

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2. *Linear DNS*
Approaches

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3. Frozen Flow Analysis
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Approaches

1. Floquet Theory
Floquet Theory

- *Floquet* mode approximation:

\[ u(r, \theta, z, \tau) \sim \hat{u}(z, \tau)e^{i\alpha r}e^{i\mu \tau}e^{in\theta} \]

- \( \hat{u}(z, \tau) \) periodic
Floquet Theory

- Floquet mode approximation:
  \[ u(r, \theta, z, \tau) \sim \hat{u}(z, \tau) e^{i\alpha r} e^{i\mu \tau} e^{in\theta} \]
- \( \hat{u}(z, \tau) \) periodic
- Harmonic decomposition gives eigenvalue problem:
  \[ \sum_{k=-K}^{K} \mathcal{L}_k \{ \mu, \alpha; n, R_k, R_s, \phi \} e^{ik\tau} = 0 \]
Floquet Theory

- **Floquet mode approximation**:

  \[ u(r, \theta, z, \tau) \sim \hat{u}(z, \tau) e^{i\alpha r} e^{i\mu \tau} e^{in\theta} \]

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- Harmonic decomposition gives eigenvalue problem:

  \[ \sum_{k=-K}^{K} \mathcal{L}_k \{ \mu, \alpha; n, R_k, R_s, \varphi \} e^{ik\tau} = 0 \]

  - Specify \( \mu \) or \( \alpha \) as real and solve for the other.
Choose $R = 500$, $n = 32$ for comparison data.

$$\alpha = 0.2814 - 0.0702i$$
Recall: $u(r, \theta, z, \tau) = u(z, \tau)e^{i\alpha r}e^{in\theta}$
$R_k = 500$, $n = 32$, $U_w \in \{0, 0.1\}$, $\varphi \in [0, 100]$

Recall: $u(r, \theta, z, \tau) = u(z, \tau) e^{i\alpha r} e^{in\theta}$
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$R_k = 500$, $n = 32$, $U_w \in \{0, 0.1, 0.2\}$, $\varphi \in [0, 100]$
Recall: \( \mathbf{u}(r, \theta, z, \tau) = u(z, \tau)e^{i\alpha r}e^{in\theta} \)

\( R_k = 500, \; n = 32, \; U_w = \{0, 0.1, 0.2, 0.25\}, \; \varphi \in [0, 100] \)
Neutral Curves

Steady Neutral Curves
Neutral Curves

\[ U_w = 0.2, \, \varphi = 15 \]
Neutral Curves

\[ U_w = 0.1 \]
Neutral Curves

\[ U_w \in \{0.1, 0.2\} \]
2. **Linear DNS**
• Measure the response of the flow to a *stationary* disturbance.
Linear DNS

- Measure the response of the flow to a stationary disturbance.
- Steady case: wall motion of the form

\[ \zeta(r, \theta, \tau) = e^{-\lambda r^2} e^{in\theta} \]
Linear DNS

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- Steady case: wall motion of the form

\[ \zeta(r, \theta, \tau) = e^{-\lambda r^2} e^{in\theta} \]

\[ u(r, z = 0, \tau > T_c) \text{ - steady: } R_k = 500, n = 32 \]
• Measure the response of the flow to a \textit{stationary} disturbance.

• $\theta$ coordinate changes:

$$\theta \rightarrow \theta_0 + \int^{\tau} \Omega(\tilde{\tau})$$
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• \( \theta \) coordinate changes:

\[
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\]

• Forcing stationary with respect to modulated disk:

\[
\zeta(r, \theta_0, \tau) = e^{\lambda r^2} e^{in\theta_0} e^{in \int_{\tilde{\tau}}^{\tau} \Omega(\tilde{\tau})}
\]
• Measure the response of the flow to a stationary disturbance.

• $\theta$ coordinate changes:

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• Forcing stationary with respect to modulated disk:

$$\zeta(r, \theta_0, \tau) = e^{\lambda r^2} e^{in\theta_0} e^{in\int_{0}^{\tau} \Omega(\tilde{\tau})}$$

negligible
• Measure the response of the flow to a *stationary* disturbance.

\[ u(r, z = 0, \tau > T_c) \text{ - steady: } R_k = 500, n = 32 \]
• Measure the response of the flow to a stationary disturbance.

\[ u(r, z = 0, \tau > T_c) - \text{modulated: } R_k = 500, n = 32, U_w = 0.1, \varphi = 10 \]
• Normal mode approximation:

\[ u(r, \theta, z, \tau) = u(z, \tau)e^{i\alpha r}e^{in\theta} \]
• Normal mode approximation:

\[ u(r, \theta, z, \tau) = u(z, \tau)e^{i\alpha r} e^{in\theta} \]

• Given \( A = u(r, z = 0, \tau > T_c) \), we can calculate:

\[ \alpha_i \simeq -i \frac{\partial A}{A} \frac{\partial}{\partial r} \]
• Normal mode approximation:

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• Given \( A = u(r, z = 0, \tau > T_c) \), we can calculate:

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• \(-\alpha_i\) gives radial growth rate.
\[
\Delta \alpha_i = \alpha_i^m - \alpha_i^s \text{ against } \varphi \text{ for } R_k = 500, \ n = 32 \text{ and } U_w \in \{0.1, 0.2\}.
\]
DNS (dots) vs. Floquet (line)
Current & Future Work

- Look at parallels between oscillation and surface roughness.
  - *S. Garrett, P. Thomas et. al.* show similar stabilising effects.
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- Experimental confirmation. [P. Thomas (Warwick)]
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- Look at parallels between oscillation and surface roughness.
  - *S. Garrett, P. Thomas et. al.* show similar stabilising effects.
- Experimental confirmation. [*P. Thomas (Warwick)*]
- Explore torsional oscillations.
Thank You