A Coupled Adjoint-Based Method for Aeroelastic Design in the Open-Source SU2 Suite

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Optimal Aeroelastic Design

Advances in computational methods and tools have made it feasible to solve fully-coupled aerodynamic and structural problems during analysis stages.

However, we want to take this further. We want to design.
1. Outline

A **brief summary** of my talk:

- **SU2 Suite**
  A high-fidelity solver for multi-disciplinary analysis and design.

- **Aeroelastic Design with SU2**
  Our work at Imperial College.

- **What’s next?**
  Bringing SU2 Aeroelastic capabilities to the next level.
Why SU2?

➢ A state-of-the-art Solver
➢ Formulated for Optimal Design
➢ Under Active Development
➢ Big Open-Source Community
A state-of-the-art **Solver**

- C++/MPI core, HPC ready
- Finite Volume, Unstructured, Multigrid
- Compressible RANS
- Aeroacoustics
- Chemically reacting flows
- Turbomachinery
- Solid mechanics FEA
- & More: High-Order, DDES, Incompressible flows, Heat transfer...

Acknowledgements to the SU2 community for the images
Formulated for Optimal Design

→ Python-wrapped core: run SciPy/pyOpt optimisers

→ Continuous & Discrete Adjoint formulation: efficient gradient calculation and extension to coupled solvers

→ Successfully applied in aerodynamic shape optimization
Under Active Development

➔ International development team in academia and industry
➔ 100+ branches
➔ ~300 active forks
➔ ~200k lines of code
➔ 1000+ commits since January
➔ Annual Developers Meeting
  December 18-19, 2017, Stanford, CA
Register Online! - https://su2code.github.io/
Big Open-Source Community

- Released on GitHub under LGPL 2.1
  
  https://github.com/su2code/SU2

- Flexible and **customizable**

- Managed repository, guarantees the quality of the code

~11000 emails on user list

~600 repo clones / month

~2000 repo visits / month

Web hits at SU2 homepage by city.
Aeroelastics Team at Imperial

Led by Dr. Rafael Palacios

Our focus is in computational methods for the multidisciplinary analysis, design and optimisation of next-generation air vehicles and wind turbines.

We target the integration of aero-structural analysis, rigid body dynamics and flight control systems.
Involvement in SU2

- **Oct 2014**: Collaboration with Stanford team starts
- **June 2016**: We lead multi-team paper at ECCOMAS
- **July 2016**: Collaboration with TU Kaiserslautern team starts
- **Dec 2017**: Full code release

2014

- **July 2015**: Implemented Non-linear FEA Solver
- **Oct 2015**: Implemented Coupled FSI Solver

2016

- **June 2016**: FSI code official release
- **Oct 2016**: Implemented Coupled Adjoint

2017

- **Sept 2017**: Paper on Coupled Adjoint @ IJNME
Coupled FSI Solver

Fluid Solver → INTERFACE → Non-Linear Structural Solver → INTERFACE → Mesh Solver

Original SU2 Solver

SU2 with FSI Solver
Coupled FSI Solver

- Fully Implicit, coupled solver
- ALE Formulation
- Geometrical Non-Linearities
- Complex material model
- Consistent interpolation on domain-filling discretizations
- Elastic solver for mesh movement
- Fully differentiated using AD
Aeroelastic Analysis

Define Input Parameters

Coupled FSI Solver

Evaluate Performance

Define Input Parameters

Coupled FSI Solver

Evaluate Performance

Fluid Solver

Mesh Solver

Structural Solver

INTERFACE

x1
x2
...
xn

Evaluate
Performance

J
Gradient-Based Optimal Aeroelastic Design

Design Performance → Compute Gradients → New Design → New Design’s Performance → Optimal Design

\[ J,0 \]
\[ \frac{dJ}{dx_1} \]
\[ \frac{dJ}{dx_2} \]
\[ \ldots \]
\[ \frac{dJ}{dx_n} \]

\[ J,1 \]
\[ x_{1,1} \]
\[ x_{2,1} \]
\[ \ldots \]
\[ x_{n,1} \]

\[ J \]
Gradient-Based Optimal Aeroelastic Design

Design Performance → Compute Gradients → Large Number of Design Variables

\[ \frac{dJ}{dx_1}, \frac{dJ}{dx_2}, \ldots, \frac{dJ}{dx_n} \]
Gradient-Based Optimal Aeroelastic Design

Design Performance

Compute Gradients

Finite Differences, 1st order
Gradient-Based Optimal Aeroelastic Design

\[ \frac{dJ}{dx_1}, \frac{dJ}{dx_2}, \ldots, \frac{dJ}{dx_n} \]

Finite Differences, 2nd order
Gradient-Based Optimal Aeroelastic Design

\[ J, 0 \]

\[ \frac{dJ}{dx_1} \]
\[ \frac{dJ}{dx_2} \]
\[ \ldots \]
\[ \frac{dJ}{dx_n} \]

**Design Performance**  
**Compute Gradients**  
**Coupled Adjoint Method**

- Fluid Adjoint
- Mesh Adjoint
- Structural Adjoint

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How do we compute Coupled Adjoint?
Computation of Coupled Adjoints...

\[
\min \ J(u, w, z, \alpha) \quad \iff \quad \text{Design Objective}
\]

subject to
\[
\begin{align*}
  u &= S(u, w, z, \alpha), \quad \iff \quad \text{Structural Problem} \\
  w &= F(w, z, \alpha), \quad \iff \quad \text{Fluid Problem} \\
  z &= M(u, \alpha). \quad \iff \quad \text{Fluid mesh Problem}
\end{align*}
\]

\[
L(\alpha, u, \bar{u}, w, \bar{w}, z, \bar{z}) = J(u, w, z, \alpha) + \bar{u}^T[S(u, w, z, \alpha) - u] \\
+ \bar{w}^T[F(w, z, \alpha) - w] + \bar{z}^T[M(u, \alpha) - z].
\]

\[
\frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} + \bar{u}^T \frac{\partial S}{\partial \alpha} + \bar{w}^T \frac{\partial F}{\partial \alpha} + \bar{z}^T \frac{\partial M}{\partial \alpha}.
\]

Gradient \iff Adjoint Variables
Computation of Coupled Adjoint...

\[
\tilde{u}^T = \frac{\partial J}{\partial u}(u, w, z, \alpha) + \tilde{u}^T \frac{\partial S}{\partial u}(u, w, z, \alpha) + \tilde{z}^T \frac{\partial M}{\partial u}(u),
\]

\[
\tilde{w}^T = \frac{\partial J}{\partial w}(u, w, z, \alpha) + \tilde{w}^T \frac{\partial F}{\partial w}(w, z, \alpha) + \tilde{u}^T \frac{\partial S}{\partial w}(u, w, z, \alpha),
\]

\[
\tilde{z}^T = \frac{\partial J}{\partial z}(u, w, z, \alpha) + \tilde{w}^T \frac{\partial F}{\partial z}(w, z, \alpha) + \tilde{u}^T \frac{\partial S}{\partial z}(u, w, z, \alpha).
\]

\[
\text{Gradient} \quad \Rightarrow \quad \frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} + \tilde{u}^T \frac{\partial S}{\partial \alpha} + \tilde{w}^T \frac{\partial F}{\partial \alpha} + \tilde{z}^T \frac{\partial M}{\partial \alpha}.
\]
...based on Algorithmic Differentiation (AD)

\[ \tilde{u}^T = \left( \frac{\partial J}{\partial u} (u, w, z, \alpha) + \tilde{u}^T \frac{\partial S}{\partial u} (u, w, z, \alpha) \right) + \tilde{z}^T \frac{\partial M}{\partial u} (u), \]

Structural-Only Term

Crossed Dependency

\[ \text{Structural Adjoint} \]

Read FSI Solution

\[ u^*, w^*, z^* \]

Define the Input

\[ \partial / \partial u \]

Iterate Solver

\[ u^* + \Delta u^* = S(u^*, w^*, z^*, \alpha) \]

Define the Output

\[ u = S(u) \]

Compute the Design Objective \( J \)

\[ J(u^* + \Delta u^*, \alpha) \]
What do we mean by **RECORDING**?

- Variable $x_1$
- Operator $\cos$
  - $\cos(x_1)$

- Variable $x_2$
- Operator $\times$
  - $x_2 \times \cos(x_1)$

**Solution**

$$y = x_2 \times \cos(x_1)$$
What do we mean by **RECORDING**?

![Diagram](image)

**Variable**
- $x_1$

**Operator**
- $\cos$
- $\cos(x_1)$

**Solution**
- $y = x_2 \cdot \cos(x_1)$
Reverse Iteration

\[
\bar{u}^T = \frac{\partial J}{\partial u}(u, w, z, \alpha) + \bar{u}^T \frac{\partial S}{\partial u}(u, w, z, \alpha) + \bar{z}^T \frac{\partial M}{\partial u}(u)
\]

- **Structural Adjoint**
- **Structural-Only Term**
- **Crossed Dependency**

**Obtain Coupled Adjoint**
- \( \bar{u} \)

**Reverse Iteration of Solver**
- \( \bar{u}_{k+1} = \bar{S}(\bar{u}_k) + \bar{u}_z \)

**Set Output Adjoint**
- \( \bar{u}_k \)

**Initialize Adjoint of Design Objective \( J \)**
- \( \bar{J} = 1.0 \)

**Incorporate Source Term**
- \( \bar{u}_z \)
Comparative of Coupled Adjoint Methods

Conventional Adjoint Methods

✗ Require exact linearisation of the primal FSI solver and sub-solvers
✗ Require analytical derivation of all the cross-discipline terms
✗ Require constant updates due to extensions of the code

AD-based Adjoint Methods

✔ Exact linearisation no longer needed, approximate Jacobians are acceptable (non-linear problems!)
✔ Automatically accommodates crossed terms and new features
✗ Require native FSI implementation
✔ Implemented in SU2!
✔ Efficient AD tools using Expression Templates
Further details...

➢ This method is fully described in a recent paper, published in the *Int J Numer Meth Engng*

➢ [doi.org/10.1002/nme.5700](https://doi.org/10.1002/nme.5700)

➢ Source code to be released in December
Applications of AD-based Coupled Adjoints

Electro-mechanically Actuated Membrane Wings

Optimal Actuation of Dielectric Membrane Wings using High-Fidelity Fluid-Structure Modelling

AIAA paper 2017-0857, presented at Scitech 2017

- Large Deformations
- Non-linear Material Model
- Electric effects
- Viscous flow
Optimal Material Distribution in Very Flexible Structures

- Large Deformations
- Non-linear Material Model
- Large number of Design Variables
- Viscous flow
Application to Wing Aeroelasticity

Imperial College Team

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Xabier Romero Sáenz, MSc Thesis, 2016
Joel Ho Munn Onn, MSc Thesis, 2017
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